

UNIVERSITY OF CALCUTTA

**SYLLABUS
FOR
FOUR -YEAR (EIGHT-SEMESTER) HONOURS AND
HONOURS WITH RESEARCH COURSE WITH
MATHEMATICS MAJOR
UNDER CURRICULUM AND CREDIT FRAMEWORK
(Semester-1 to Semester-6)**

and

**SYLLABUS
FOR
THREE -YEAR (SIX-SEMESTER)
MULTIDISCIPLINARY COURSE WITH MATHEMATICS
(Semester-1 to Semester-6)**

Odd Semester: July to December

Even Semester: January to June

The syllabus for the 4 Year Honours and Honours with Research Course with Mathematics Major is effective from the academic year **2023-2024**.

The syllabus for the 3 Year Multidisciplinary Course with Mathematics is effective from the academic year **2023-2024**.

Semester One					
Course	Paper	Paper code	Name of Paper	Credit	Page No.
MAJOR	DSCC 1	MTHM	Calculus, Geometry and Vector Analysis	3(TH)+1(TU)=4	5
MAJOR	SEC 1	MTHM	C language with Mathematical Applications	3(TH)+1(TU)=4	8
MINOR	MN1	MMTH	Calculus, Geometry and Vector Analysis	3(TH)+1(TU)=4	6
MDC	CC 1	MMTH-MDC	Calculus, Geometry and Vector Analysis	3(TH)+1(TU)=4	6
MDC	SEC	MMTH-SEC	C language with Mathematical Applications	3(TH)+1(TU)=4	8
IDC	IDC	MTHD	Mathematics in Daily Life	2(TH)+1(TU)=3	15

Semester Two					
Course	Paper	Paper code	Name of Paper	Credit	Page No.
MAJOR	DSCC 2	MTHM	Basic Algebra	3(TH)+1(TU)=4	7
MAJOR	SEC 2.1	MTHM	Python Programming and Introduction to Latex	3(TH)+1(TU)=4	10
MAJOR	SEC 2.2	MTHM	Artificial Intelligence	3(TH)+1(TU)=4	
MINOR	MN2	MMTH	Basic Algebra	3(TH)+1(TU)=4	7
MDC	CC 2	MMTH-MDC	Basic Algebra	3(TH)+1(TU)=4	7
MDC	SEC	MMTH-SEC	C language with Mathematical Applications	3(TH)+1(TU)=4	8
IDC	IDC	MTHD	Mathematics in Daily Life	2(TH)+1(TU)=3	15

In Semester II either to study **Python Programming and Introduction to Latex** or **Artificial Intelligence**.

Semester Three					
Course	Paper	Paper code	Name of Paper	Credit	Page No
MAJOR	DSCC 3	MTHM	Real Analysis	3(TH)+1(TU)=4	19
MAJOR	DSCC 4	MTHM	Ordinary Differential equations-I and Group Theory-I	3(TH)+1(TU)=4	21
MAJOR	SEC 3	MTHM	Linear Programming and Rectangular Games	3(TH)+1(TU)=4	14
MINOR	MN3	MMTH	Calculus, Geometry and Vector Analysis	3(TH)+1(TU)=4	6
MDC	CC3	MMTH-MDC	Ordinary Differential Equations and Group Theory	3(TH)+1(TU)=4	21
MDC	SEC	MMTH-SEC	C language with Mathematical Applications	3(TH)+1(TU)=4	8
MDC	MDC-mn1	MMTH-MDC-mn	Calculus, Geometry and Vector Analysis	3(TH)+1(TU)=4	6
IDC	IDC	MTHD	Mathematics in Daily Life	2(TH)+1(TU)=3	15

Semester Four					
Course	Paper	Paper code	Name of Paper	Credit	Page No
MAJOR	DSCC 5	MTHM	Theory of Real Functions	3(TH)+1(TU)=4	23
MAJOR	DSCC 6	MTHM	Mechanics-I	3(TH)+1(TU)=4	24
MAJOR	DSCC 7	MTHM	Multivariate Calculus-I&Partial Differential Equations-I	3(TH)+1(TU)=4	26
MAJOR	DSCC 8	MTHM	Group Theory-II and Ring Theory-I	3(TH)+1(TU)=4	28
MINOR	MN4	MMTH	Basic Algebra	3(TH)+1(TU)=4	7
MDC	CC 4	MMTH-MDC	Mechanics	3(TH)+1(TU)=4	24
MDC	CC 5	MMTH-MDC	Advanced Calculus	3(TH)+1(TU)=4	44
MDC	MDC-mn2	MMTH-MDC-mn	Basic Algebra	3(TH)+1(TU)=4	7

Semester Five					
Course	Paper	Paper code	Name of Paper	Credit	Page No.
MAJOR	DSCC 9	MTHM	Probability and Statistics	3(TH)+1(TU)=4	30
MAJOR	DSCC 10	MTHM	Ring Theory-II and Linear Algebra-I	3(TH)+1(TU)=4	32
MAJOR	DSCC 11	MTHM	Riemann Integration and Series of function	3(TH)+1(TU)=4	34
MAJOR	DSCC 12	MTHM	Mechanics-II	3(TH)+1(TU)=4	36
MINOR	MN5	MMTH	Ordinary Differential equations-I & Group Theory-I	3(TH)+1(TU)=4	21
MDC	CC 6	MMTH-MDC	Statistics and Numerical Analysis	3(TH)+1(TU)=4	46
MDC	*CC 7	MMTH-MDC	Mathematical Methods	3(TH)+1(TU)=4	49
MDC	MDC-mn3	MMTH-MDC-mn	Ordinary Differential Equations and Group Theory	3(TH)+1(TU)=4	21
MDC	MDC-mn4	MMTH-MDC-mn	Mechanics	3(TH)+1(TU)=4	24

*paper CC7 shall be studied either in semester-5 (if opted as Core Course-1) or semester-6 (if opted as Core Course-2)

Semester Six					
Course	Paper	Paper code	Name of Paper	Credit	Page No.
MAJOR	DSCC 13	MTHM	Metric Space & Complex Analysis-I	3(TH)+1(TU)=4	38
MAJOR	DSCC 14	MTHM	Multivariate Calculus-II and Application of Calculus	3(TH)+1(TU)=4	40
MAJOR	DSCC 15	MTHM	Numerical Analysis	3(TH)+1(PR)=4	42
MINOR	MN6	MMTH	Mechanics	3(TH)+1(TU)=4	24
MDC	CC 8	MMTH-MDC	Discrete Mathematics	3(TH)+1(TU)=4	51
MDC	MDC-mn5	MMTH-MDC-mn	Advanced Calculus	3(TH)+1(TU)=4	44
MDC	MDC-mn6	MMTH-MDC-mn	Statistics and Numerical Analysis	3(TH)+1(TU)=4	46

Note:

- 1.If Mathematics is chosen as Core Course 2 in MDC then only paper CC6 [Statistics and Numerical Analysis (MATH-MD-CC6-5-TH)] will be taught in semester five and the papers CC7[Application of Calculus and Advanced Algebra (MATH-MD-CC7-6-TH)] & CC8[Discrete Mathematics(MATH-MD-CC-8-6-TH)] will be taught in semester six.
2. Syllabus of **Ordinary Differential Equations – I and Group Theory – I** and **Ordinary Differential Equations and Group Theory** is same.
3. Syllabus of **Mechanics-I** and **Mechanics** is same.
4. To get syllabus and question pattern of **Artificial Intelligence** see Notification No. CSR/36/2023 and Notification No. CSR/35/2024.
5. Tutorial marks will be awarded based on internal assessment– by evaluation of internal assignments for SEC papers and by internal examination for Core, Minor, IDC papers.

MATH-H-CC1-1-TH**Calculus, Geometry & Vector Analysis**

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A: Calculus

[Marks:20] [16 classes]

- Differentiability of a function at a point and in an interval. Meaning of sign of derivative. Differentiating hyperbolic functions, higher order derivatives, Leibnitz rule and its applications to functions of type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax + b)^n \sin x$, $(ax + b)^n \cos x$. Indeterminate forms. L'Hospital's rule (statement and example).
- Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x dx$, $\int \cos^n x dx$, $\int \tan^n x dx$, $\int \sec^n x dx$, $\int (\log x)^n dx$, $\int \sin^n x \sin^m x dx$, $\int \sin^n x \cos^m x dx$. Parametric equations, parametrizing a curve, arc length of a curve, arc length of parametric curves, area under a curve, area and volume of surface of revolution.

Group B: Geometry

[Marks:35] [28 classes]

- Rotation of axes and second degree equations, classification of conics using the discriminant, reduction to canonical form, tangent and normal, polar equations of conics.
- Spheres. Cylindrical surfaces. Central conicoids, paraboloids, plane sections of conicoids, generating lines, identification of quadric surfaces like cone, cylinder, ellipsoid, hyperboloid, classification of quadrics.

Group C: Vector Analysis

[Marks: 20] [16 classes]

- Triple product, vector equations, applications to geometry and mechanics — concurrent forces in a plane, theory of couples, system of parallel forces. Introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions of one variable.

References:

- [1] G.B. Thomas and R.L. Finney, Calculus, 14th Ed., Pearson Education, Delhi, 2018.
- [2] M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) P. Ltd. (Pearson Education), Delhi, 2022.
- [3] H. Anton, I. Bivens and S. Davis, Calculus, 10th Ed., John Wiley and Sons (Asia) P. Ltd., Singapore, 2015.
- [4] R. Courant and F. John, Introduction to Calculus and Analysis (Volumes I & II), Springer- Verlag, New York, Inc., 1998.
- [5] T. Apostol, Calculus, Volumes I and II, Wileyand Sons, 1969
- [6] R. R. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing, 2020.
- [7] Marsden, J., and Tromba, Vector Calculus, W. H. Freeman & Co., 6th edition, 2011.
- [8] M.R. Spiegel, Schaum's outline of Vector AnalysisTata McGraw Hill Ed., 2011.
- [9] S. L. Loney, Co-ordinate Geometry, 6th Edition, Arihant Publications, 2016.
- [10] Robert J. T. Bell, Co-ordinate Geometry of Three Dimensions, Macmillan and Co., Ltd., London, 2018.

MATH-H-CC2-2-TH

Basic Algebra

Full Marks: 100 (Theory: 75 and Tutorial:25)

Group A

[Marks:25] [20 classes]

- Polar representation of complex numbers, n^{th} roots of unity, De Moivre's theorem for rational indices and its applications. Exponential, logarithmic, trigonometric and hyperbolic functions of complex variable.
- Theory of equations: Relation between roots and coefficients, transformation of equation, Descartes rule of signs, Application of Sturm's theorem, cubic equation (solution by Cardan's method) and biquadratic equation (solution by Ferrari's method).
- Inequalities: The inequality involving $AM \geq GM \geq HM$, Cauchy-Schwartz inequality.

Group B

[Marks: 25] [20 classes]

- Relation: equivalence relation, equivalence classes & partition, partial order relation, poset, linear order relation.
- Mapping: composition of mappings, relation between composition of mappings and various set theoretic operations. Meaning and properties of $f^{-1}(B)$, for any mapping $f : X \rightarrow Y$ and $B \subseteq Y$.
- Well-ordering property of positive integers, Principles of Mathematical induction, equivalence of Wellordering property and Principles of Mathematical induction (statement only), division algorithm, divisibility and Euclidean algorithm. Prime numbers and their properties, Euclid's theorem. Congruence relation between integers. Fundamental Theorem of Arithmetic. Chinese remainder theorem. Arithmetic functions, some arithmetic functions such as ϕ , τ , σ and their properties.

Group C

[Marks:25] [20 classes]

- Systems of linear equations, homogeneous and non-homogeneous systems. Existence and Uniqueness of solution. The matrix equation $Ax = b$, row reduction and echelon forms, uniqueness of reduced echelon form. Rank of a matrix and characterization of invertible matrices, Pivot positions, basic and free variables, parametric description of the solution set. Existence and uniqueness theorem.
- Vectors in R^n , algebraic and geometric properties of the vectors. Vector form of a linear system and the column picture. Existence of solutions and linear combination of vectors. Geometry of linear combination and subsets spanned by some vectors.

Uniqueness of solution and linear independence of vectors. Algebraic and geometric characterizations of linearly independent subsets.

References

- [1] Titu Andreescu and Dorin Andrica, Complex Numbers from A to Z, 2nd Ed., Springer Nature, 2014.
- [2] Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory, 3rd Ed., Pearson Education (Singapore) P. Ltd., Indian Reprint, 2005.
- [3] David C. Lay, Linear Algebra and its Applications, 3rd Ed., Pearson Education Asia, Indian Reprint, 2007.
- [4] Gilbert Strang; Introduction to Linear Algebra (5th Edition); Wellesley-Cambridge Press, 2019.
- [5] Anton Howard and Chris Rorres; Elementary Linear Algebra with Supplemental Applications (11th Edition); Wiley, 2014.
- [6] K. Hoffman, R. Kunze, Linear algebra, Prentice Hall India Learning Pvt. Ltd., 2015.
- [7] W.S. Burnside and A.W. Panton, Theory of equations, Dublin University Press Series, S. Chand and Company Pvt. Ltd., 1986.

MATH-H-SEC1-1-TH

C Language with Mathematical Applications

Full marks: 100
(Theory: 75 and Tutorial: 25)
(60classes)

Overview of architecture of computer, compiler, assembler, machine language, high level language, object oriented language, programming language, higher level language

- Constants, Variables and Data type of C-Program: Character set. Constants and variables data types, expression, assignment statements, declaration.
- Operation and Expressions: Arithmetic operators, relational operators, logical operators.
- Decision Making and Branching: decision making with if statement, if-else statement, Nesting if statement, switch statement, break and continue statement.
- Control Statements: While statement, do-while statement, for statement.

- Arrays: One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.
- User-defined Functions: Definition of functions, Scope of variables, return values and their types, function declaration, function call by value, Nesting of functions, passing of arrays to functions, Recurrence of function.
- Introduction to Library functions: stdio.h, math.h, string.h, stdlib.h, time.h etc.

Sample problems:

1. Display first 15 natural numbers.
2. Compute the sum of first 10 natural numbers.
3. Read 10 numbers from keyboard and find their average.
4. Find the sum of first 15 even natural numbers.
5. Write a program to find factorial of a number using recursion.
6. Write a program to make a pyramid pattern with numbers increased by 1.
7. From the terminal read three values, namely, length, width, height. Print a message whether the box is a cube or rectangle or semi-rectangle.
8. Find the AM, GM, HM of a given set of numbers.
9. Write a program to print multiplication table.
10. Write a program that generates a data file containing the list of customers and their contact numbers.
11. Find the maximum and minimum element of a given array.
12. Sort the elements of an array in ascending order
13. Write a program to read in an array of names and to sort them in alphabetical order.
14. Write a program for addition of two matrices.
15. Find the transpose of a given matrix.
16. Find the product of two matrices.
17. Write a program to check whether two given strings are an anagram.
18. Write a program to check Armstrong and Perfect numbers.
19. Write a program to check whether a number is a prime number or not.
20. Prepare a code for summing a Series.
21. Compute approximate value of pi .
22. Compute the area under a given curve.
23. Solve a quadratic equation.
24. Write a program to solve a system of two linear equations in two unknowns.
25. Write a program to find the shortest distance between two straight lines (parallel or intersecting or skew) in space.

26. Prepare an investment report by calculating compound interest.

Note: A practical note book is to be prepared with the internal assignments and to be submitted for the partial fulfilment of the course.

References

- [1] B. W. Kernighan and D. M. Ritchi : The C-Programming Language, 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.
- [2] E. Balagurnsamy : Programming in ANSI C, Tata McGraw Hill, 2004.
- [3] Y. Kanetkar : Let Us C ; BPB Publication, 1999.
- [4] C. Xavier : C-Language and Numerical Methods, New Age International, 2007.
- [5] V. Rajaraman : Computer Oriented Numerical Methods, Prentice Hall of India, 1980

MATH-H-SEC 2.1-2-TH

Python Programming and Introduction to Latex

Full marks: 100
(Theory: 75 and Tutorial: 25)

Group A: Python Programming

[Marks: 50][40 classes]

Python Programming Language, features, Installing Python. Running Code in the Interactive Shell, IDLE. Input, Processing and Output, Editing, Saving, and Running a Script, Debugging: Syntax Errors, Runtime Errors, Semantic Errors.

Data types and expressions: Variables and the Assignment Statement, Program Comments and Doc strings. Data Types-Numeric integers and Floating-point numbers. Boolean string. Mathematical operators, PEMDAS.Arithmetic expressions, Mixed-Mode Arithmetic and type Conversion, type(). Input(), print(), program comments. id(), int(), str(), float().

Loops and selection statements: Definite Iteration: for Loop, Executing statements a given number of times, Specifying steps using range(), Loops that count down, Boolean and Comparison operators and Expressions, Conditional and alternative statements- Chained and Nested Conditionals: if, if-else, if-

elseif-else, nested if, nested if-else. Compound Boolean Expressions, Conditional Iteration: while Loop –with True condition, break Statement. Random Numbers. Loop Logic, errors and testing.

Strings, Lists, Tuple, Dictionary: Accessing characters, indexing, slicing, replacing. Concatenation (+), Repetition (*). Searching a substring with the 'in' Operator, Traversing string using while and for. String methods- find, join, split, lower, upper. len().

Lists – Accessing and slicing, Basic Operations (Comparison, +), List membership and for loop. Replacing element (list is mutable). List methods- append, extend, insert, pop, sort. Max(), min(). Tuples. Dictionaries- Creating a Dictionary, Adding keys and replacing Values , dictionary - key(), value(), get(), pop(), Traversing a Dictionary. Math module: sin(), cos(), exp(), sqrt(), constants- pi, e.

Design with functions: Defining Simple Functions- Parameters and Arguments, the return Statement, tuple as return value. Boolean Functions. Defining a main function. Defining and tracing recursive functions.

Working with Numbers: Calculating the Factors of an Integer, Generating Multiplication Tables, converting units of measurement, Finding the roots of a quadratic equation

Algebra and Symbolic Math with SymPy: symbolic math using the SymPy library. Defining Symbols and Symbolic Operations, factorizing and expanding expressions, Substituting in Values, Converting strings to mathematical expressions. Solving equations, Solving quadratic equations, Solving for one variable in terms of others, Solving a system of linear equations.

Plotting using SymPy, Plotting expressions input by the user, Plotting multiple functions

Sample problems:

1. Convert number from decimal to binary system.
2. Convert number from decimal to octal system.
3. Convert from Hexadecimal to binary system.
4. Write a program to read one subject mark and print pass or fail. Use single return values function with argument.
5. Find the median of a given set of numbers.

6. Write a Python function that takes two lists and returns True if they have at least one common member.
7. Write a program for Enhanced Multiplication Table Generator.
8. Write down Unit converter code.
9. Write down Fraction Calculator code.
10. Write down Factor Finder code.
11. Write down Graphical Equation Solver code.
12. Write down a code for solving Single-Variable Inequalities.
13. Prepare an investment report by calculating compound interest.
14. Write a python program to open and write the content to file and read it.
15. Write a python program to check whether a given year is leap year or not and also print all the months of the given year.

Group B: Introduction to Latex

[Marks: 25] [20 classes]

Introduction to LATEX: Preparing a basic LATEX file. Compiling LATEX file.

Document classes: Different type of document classes, e.g., article, report, book etc.

Page Layout: Titles, Abstract, Chapters, Sections, subsections, paragraph, verbatim, References, Equation references, citation.

List structures: Itemize, enumerate, description etc.

Representation of mathematical equations: Inline math, Equations, Fractions, Matrices, trigonometric, logarithmic, exponential functions, line, surface, volume integrals with and without limits, closed line integral, surface integrals, Scaling of Parentheses, brackets etc.

Customization of fonts: Bold fonts, emphasise, `mathbf`, `mathcal` etc. Changing sizes Large, Larger, Huge, tiny etc.

Writing tables: Creating tables with different alignments, placement of horizontal, vertical lines.

Figures: Changing and placing the figures, alignments

Packages: amsmath,amssymb, graphics, graphicx, Geometry, algorithms, color, Hyperref etc. Use of Different LATEX commands and environments, Changing the type style, symbols from other languages. special characters.

Sample Projects:

1. Write down a research article.
2. Write down a given mathematical derivation.
3. Write a book chapter.
4. Write a report on a practical done in laboratory with results, tables and graphs.
5. Present graphical analysis taking graphs plotted in gnuplot.

Note: A practical note book is to be prepared with the internal assignments and to be submitted for the partial fulfilment of the course.

References

- [1] Kenneth A Lambert, Fundamentals of Python: First programs, 2nd edition – Cengage Learning India, 2019.
- [2] Saha Amit, Doing Math with Python - No starch press, San Francisco, 2015.
- [3] E. Balgurusamy, Problem solving and Python programming- Tata McGraw Hill, 2017.
- [4] LATEX- A Document Preparation System, Leslie Lamport, Addison-Wesley, 1994.
- [5] E. Krishnan, LATEXTutorials A PRIMER, Indian TEXusers group, 2003.
- [6] George Gratzner, Practical LATEX, Springer, 2014.

MATH-H-SEC3-3-TH

Linear Programming and Rectangular Games

Full Marks: 100 (Theory : 75 marks and Tutorial: 25 marks)
(60classes)

- Definition of Linear Programming Problem (L.P.P.). Formation of L.P.P. from daily life involving inequations. Graphical solution of L.P.P. Basic solutions and Basic Feasible Solution (B.F.S) with reference to L.P.P. Matrix formulation of L.P.P. Degenerate and Non-degenerate B.F.S.
- Hyperplane, Convex set, Cone, extreme points, convex hull and convex polyhedron. Supporting and Separating hyperplane. The collection of a feasible solutions of an L.P.P. constitutes a convex set. The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions (the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L.P.P. admits of an optimal solution then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S.
- Slack and surplus variables. Standard form of L.P.P. theory of simplex method. Feasibility and optimality conditions. Algorithm. Two phase method. Degeneracy in L.P.P. and its resolution.
- Duality theory: The dual of dual is the primal. Relation between the objective values of dual and the primal problems. Relation between their optimal values.

Post-optimal Analysis: Discrete changes in the cost vector, Discrete changes in the requirement vector, Discrete changes in the coefficient matrix, Addition of a variable, Addition of a constraint.

- Transportation and Assignment problems. Mathematical justification for optimality criterion. Hungarian method. Traveling Salesman problem.
- Concept of game problem. Rectangular games. Pure strategy and Mixed strategy. Saddle point and its existence. Optimal strategy and value of the game. Necessary and sufficient condition for a given strategy to be optimal in a game. Concept of Dominance. Fundamental Theorem of rectangular games. Algebraic method. Graphical method and Dominance method to solve Rectangular games. Inter-relation between theory of games and L.P.P.

Note:1. Students will learn formulation of L.P.P. and obtaining optimal solution of L.P.P. using software package.

2. A practical note book is to be prepared with the internal assignments and to be submitted for the partial fulfilment of the course.

References

- [1] Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, Linear Programming and Network Flows, 2nd Ed., John Wiley and Sons, India, 2004.
- [2] F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, 9th Ed., Tata McGraw Hill, Singapore, 2009.
- [3] Hamdy A. Taha, Operations Research, An Introduction, 8th Ed., Prentice-Hall India, 2006.
- [4] G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.
- [5] Churchman, Ackoff, Arnoff, Introduction to Operations Research, John Wiley and Sons Inc., 1957.
- [6] Billy, E. Gillet, Introduction to Operations Research: A Computer Oriented Algorithmic Approach, TMH Edition, 1979.
- [7] Swarup K., Gupta P.K., Man Mohan, Operations Research, Sultan Chand and Sons, 2020.
- [8] Chakraborty J. G. and Ghosh, P.R., Linear Programming and Game Theory, MoulikLibrary, 1979.

MATH-H-IDC-1-TH **Mathematics in DailyLife**

Full marks: 75 (Theory: 50 and Tutorial: 25)
(45classes)

Group A: Basics of Set Theory

[Marks: 4][4 classes]

- Concept and definition of sets, subsets and set operations (Union, Intersection, Complementation, Subtraction); Statements of basic laws of set algebra.

- Venn diagrams. Statement of the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and its application in daily life.

Group B: Understanding Integers

[Marks: 20][18 classes]

- Statement and simple problems on First Principle of Mathematical Induction.
- Statement of Division algorithm; G.C.D. of two positive integers, Expression of G. C. D. of two integers x, y in the form $px + qy$ (p, q are integers), (Euclidean Algorithm without proof).
- Representation of a positive integer in Binary and decimal mode.
- Linear Diophantine equation in two variables: Statement of condition on the existence of integral solution, General / particular solution, Simple real life applications;
- Prime Integers. Some elementary properties of prime integers (only statement), Fundamental theorem of Arithmetic (only statement), Algorithm for Primality test.
- Congruence of Integers: Meaning of $a \equiv b \pmod{m}$, Statements of elementary properties of congruence; If $a \equiv b \pmod{m}$ then for any integer c , $(a + c) \equiv (b + c) \pmod{m}$, $(a - c) \equiv (b - c) \pmod{m}$, $ac \equiv bc \pmod{m}$, $a^n \equiv b^n \pmod{m}$ for natural numbers n ;
- Application of congruence of integers: Divisibility tests by 2, 3, 4, 5, 7, 9, 11, 13 (Statements of relevant results and problems only), Check Digits in International Standard Book Number (ISBN), Universal Product Code (UPC), VISA and MASTER card (Statements of relevant results and Problems only), Formation of Round Robin Tournament Table using congruence of integers(Technique and Problems only).

Group C:Mathematical Logic

[Marks: 7][6 Classes]

- Proposition, propositional variables and propositional Logic;
- Logical Connectives: NOT (Negation), OR (Disjunction), AND (Conjunction), Exclusive OR(XOR), IMPLICATION(If p then q) and BI-IMPLICATION (If and only if) and their Truth Tables; Truth value of a proposition, Truth tables of expressions involving more than one logical connective;
- Tautology, logical consequence, logical equivalence, contradiction;

Group D: Basics of Operations Research

[Marks: 9][8 classes]

- Idea of Linear Programming Problems: Objective function, decision variables, constraints.
- Formulation of daily life problems as an LPP (e.g. Carpenter problem, preparation of mixtures of chemicals, diet problems etc.);
- Solution of an LPP by graphical method.(only bounded region)
- Definition of Game, Examples from daily life Two person zero sum game, Strategy, Payoff, Saddle point, Solution of a game problem with saddle point (only elementary problems)

Group E: Financial Mathematics

[Marks: 10][9 classes]

- Time value of money:- Simple interest and Compound interest (Fundamental Formulae); Interest payable monthly, quarterly, annually; (Only problems).

- Ordinary Simple Annuities – Accumulated value and Discounted Value of an ordinary simple annuity – Idea of repayment of loans, Simple problems. (No formula derivation).
- Problems on Dividend calculation and Calculation of income tax on taxable income (old and new regime).

References:

- [1] Richard Courant and Herbert Robbins; What is Mathematics? Oxford University Press, 1995
- [2] David M. Burton; Elementary Number Theory, Universal Book Stall, 1989
- [3] Kenneth H. Rosen, Elementary Number Theory and its Applications; Addison-Wesley Publishing Company, 1984
- [4] M.K.Sen and B.C. Chakraborty; Introduction to Discrete Mathematics, Books and Allied (P) Ltd, 2019
- [5] Elliott Mendelson; Introduction to Mathematical Logic; Chapman & Hall; London, 1997
- [6] M. Chakraborty; Lecture note: A journey through the logic wonderland, IEST Shibpur, 2016
- [7] Paul R. Thie and G. E. Keough, An Introduction To Linear Programming and Game Theory; John Wiley & Sons, INC., Third Edition, 2008
- [8] [Richard Bronson](#) and [Govindasami Naadimuthu](#), Schaum's Outline of Operations Research; [McGraw Hill](#), 1997
- [9] J. G. Chakraborty and P.R. Ghosh, Linear Programming and Game Theory, Moulik Library, 2009
- [10] Petr Zima and Robert L. Brown, Mathematics of Finance, Schaum's Outline Series, McGraw-Hill, 2nd edition, 1996
- [11] P.Chandra, Investment Analysis and Portfolio Management; McGraw Hill (2008)
- [12] Bonnie Averbach and Orin Chein, Problem Solving Through Recreational Mathematics, Dover Publications, 1980.

MATH-H-CC3-3-TH

Real Analysis

Full Marks: 100 (Theory: 75 and Tutorial:25)

Group A

[Marks: 30][24 classes]

- Intuitive idea of real numbers. Mathematical operations and usual order of real numbers revisited with their properties (closure, commutative, associative, identity, inverse, distributive). Idea of countable sets, uncountable sets and uncountability of \mathbb{R} . Concept of bounded and unbounded sets in \mathbb{R} . L.U.B. (supremum), G.L.B. (infimum) of a set and their properties. L.U.B. axiom or order completeness axiom. Archimedean property of \mathbb{R} . Density of rational (and Irrational) numbers in \mathbb{R} .
- Intervals. Neighbourhood of a point. Interior point. Open set. Union, intersection of open sets. Limit point and isolated point of a set. Bolzano-Weierstrass theorem for sets. Existence of limit point of every uncountable set as a consequence of Bolzano-Weierstrass theorem. Derived set. Closed set (defined as Complement of open set). Union and intersection of closed sets as a consequence. No nonempty proper subset of \mathbb{R} is both open and closed. Expressing an open set of \mathbb{R} as countable union of disjoint open intervals (statement only). Dense set in \mathbb{R} as a set having non-empty intersection with every open interval. \mathbb{Q} and $\mathbb{R} \setminus \mathbb{Q}$ are dense in \mathbb{R} .

Group B

[Marks: 35][28 classes]

- Real sequence. Bounded sequence. Convergence and non-convergence. Examples. Boundedness of convergent sequence. Uniqueness of limit. Algebra of limits.
- Relation between the limit point of a set and the limit of a convergent sequence of distinct elements. Monotone sequences and their convergence. Sandwich rule. Nested interval theorem. Limit of some important sequences : $\left\{n^{\frac{1}{n}}\right\}_n$, $\{x^n\}_n$, $\{x^{1/n}\}_n$, $\{x_n\}_n$ with $\frac{x_{n+1}}{x_n} \rightarrow l$ and $|l| < 1$, $\left\{\left(1 + \frac{1}{n}\right)^n\right\}_n$, $\left\{1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}\right\}_n$, $\{a^{x_n}\}_n$ ($a > 0$). Cauchy's first and second limit theorems.

- Subsequence. Subsequential limits, \limsup as the L.U.B. and \liminf as the G.L.B of a set containing all the subsequential limits. Alternative definition of \limsup and \liminf of a sequence using inequality or as $\limsup x_n = \inf_n \sup\{x_n, x_{n+1}, \dots\}$ and $\liminf x_n = \sup_n \inf\{x_n, x_{n+1}, \dots\}$ [Equivalence between these definitions is assumed]. A bounded sequence $\{x_n\}_n$ is convergent if and only if $\limsup x_n = \liminf x_n$. Every sequence has a monotone subsequence. Bolzano-Weierstrass theorem for sequence. Cauchy sequence. Cauchy's general principle of Convergence.

Group C

[Marks: 10][8 classes]

- Infinite series, convergence and non-convergence of infinite series, Cauchy criterion, tests for convergence; comparison test, limit comparison test, ratio test, Cauchy's n th root test, Kummer's test (statement and problems), Raabe's test (statement and problems), Gauss test (statement and problems). Alternating series, Leibniz test. Absolute and conditional convergence, Riemann's rearrangement theorem (statement and problems).

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MATH-H-CC4-3-TH
Ordinary Differential Equations – I
and Group Theory - I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A: Ordinary Differential Equations – I

[Marks: 45][36 classes]

- Formation of differential equations, order and degree of a differential equation, First order and first degree differential equations; Homogeneous and exact differential equations, conditions for an equation of the first order to be exact, Integrating factors, Rules for finding integrating factors, Linear equations and Bernoulli equations.
- First order higher degree differential equations solvable for x , y and p , Clairaut's forms. Singular solutions, Equations of tac-locus, nodal locus, cuspidal locus.
- Higher order linear and nonlinear equations, Concept of Wronskian and its properties, Complementary functions, Particular integrals, linear homogeneous and non-homogeneous equations with constant coefficients, Method of undetermined coefficients, Method of variation of parameters. Simultaneous linear differential equations.
- Higher order linear equations with variable coefficients reducible to linear equations with constant coefficients (Euler's equation), Condition for exactness of higher order linear equations, Integrating factors, Equations of the form $\frac{d^n y}{dx^n} = f(y)(n \geq 2)$.

Group-B: Group Theory – I

[Marks: 30][24 classes]

- Definition of a group, examples of groups including permutation groups, dihedral groups and quaternion groups (through matrices), elementary properties of groups, examples of commutative and non-commutative groups. Subgroups and examples of subgroups, necessary and sufficient

condition for a nonempty subset of a group to be a subgroup, Normalizer, centralizer, center of a group, product of subgroups.

- Order of an element of a group, order of a group, cyclic group, properties of cyclic groups, classification of subgroups of cyclic groups, Permutation, cycle notation for permutations, properties of permutation, even and odd permutations, Alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's little theorem.

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- [1] S. Ahamad and A. Ambrosetti, A Textbook on Ordinary Differential Equations, Springer Verlag, 2015.
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MATH-H-CC 5-4-TH

Theory of Real Functions

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A : Limit and Continuity of Functions

[Marks: 45][36 classes]

- Limits of functions ($\epsilon - \delta$ approach), sequential criterion for limits. Cauchy's criterion of existence of limit (statement only). Limit theorems, one sided limits. Infinite limits and limits at infinity. Important limits like $\frac{\sin x}{x}$, $\frac{\log(1+x)}{x}$, $\frac{a^x-1}{x}$ ($a > 0$) as $x \rightarrow 0$.
- Continuity of a function on an interval and at an isolated point. Sequential criteria for continuity. Concept of oscillation of a function at a point. A function is continuous at x if and only if its oscillation at x is zero. Familiarity with the figures of some well known functions: $y = x^a$ ($a = 2, 3, 1/2, -1$), $|x|$, $[x]$, $\sin x$, $\cos x$, $\tan x$, $\log x$, e^x . Algebra of continuous functions as a consequence of algebra of limits. Continuity of composite functions. Examples of continuous functions. Continuity of a function at a point does not necessarily imply the continuity in some neighbourhood of that point.
- Bounded functions. Neighbourhood properties of continuous functions regarding boundedness and maintenance of same sign. Continuous function on a closed interval $[a, b]$ is bounded and attains its bounds therein. Bolzano's theorem. Intermediate value theorem.
- Discontinuity of functions, type of discontinuity. Step functions. Piecewise continuity. Monotone functions. Monotone functions can have only jump discontinuity. Monotone functions can have at most countably many points of discontinuity. Monotone bijective function from an interval to an interval is continuous and its inverse is also continuous.
- Uniform continuity. Functions continuous on a closed and bounded interval is uniformly continuous. A necessary and sufficient condition under which a continuous function on a bounded open interval I will be uniformly continuous on I . A sufficient condition under which a continuous function on an unbounded open interval I will be uniformly continuous on I (statement only). Lipschitz condition and uniform continuity.

Group B: Differentiability of Functions

[Marks: 30][24 classes]

- Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy — as an application of Rolle's theorem. Taylor's theorem on closed and bounded interval with Lagrange's and Cauchy's form of remainder. Expansion of e^x , $\log(1+x)$, $(1+x)^m$, $\sin x$, $\cos x$ with their range of validity (assuming

relevant theorems). Application of Taylor's theorem to inequalities. Point of local extremum (maximum, minimum) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point (statement only). Application of the principle of maximum/minimum in geometrical problems.

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MATH-H-CC 6-4-TH Mechanics-I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Statics-I:

- Idea about Physical Independence Principle of Forces, Principle of transmissibility of a force, Principle of action and reaction and Principle of parallelogram law of forces, Composition and resolution of forces, Concurrent Forces in a plane, Composition and resolution of forces, Equilibrium of three forces acting at a point, Lami's theorem, Moment of a force about a point and an axis, Varignon's theorem, Resultant forces and resultant couple, Coplanar forces: Its reduction and conditions of equilibrium. [8 classes]

Particle Dynamics-I:

- Law of gravitation, Concept of inertial frame, Newton's laws of motion, Concept of equation of motion of a particle, Rectilinear motion in a given force field, Simple harmonic motion, damped and forced oscillations, Concept of resonance, motion of elastic strings, Rectilinear motion under uniform gravity, Rectilinear motion in a resisting medium where resistance is proportional to velocity.[18 classes]
- Work, power, energy, Conservative forces, Potential energy, Existence of potential energy function, Conservative field and Principle of conservation of energy. [6 classes]
- Impulse of a force, Impulsive force, Principle of conservation of linear momentum, Collision of elastic bodies: Coefficient of restitution, Newton's law of collision, Direct and oblique impact of a smooth sphere with a fixed plane, Direct and oblique impact of two smooth spheres. [8 classes]
- Motion of a particle in a plane (2D Cartesian): Angular velocity and angular acceleration, Expressions for components of velocity and acceleration, Tangential and normal components of velocity and acceleration, Motion of a projectile in a resisting medium under gravity. Motion of a particle in a plane (2D Polar): Expressions for components of velocity and acceleration, Central forces and central orbits, Motion under inverse square law, Times of describing the arcs of central orbits for a particle moving under inverse square law, Kepler's laws on planetary motion, Motion of artificial satellites, Tangential and normal components of velocity and acceleration, Constrained motion of a particle on smooth curve.[20 classes]

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- [1] D. Chernilevski, E. Lavrova and V. Romanov, Mechanics for Engineers, MIR Publishers, 1984.
- [2] F. Chorlton, Textbook of Dynamics, CBS Publishers, 2002 (2nd edition).
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MATH-H-CC 7-4-TH

Multivariate Calculus – I and Partial Differential Equations – I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A: Multivariate Calculus – I

[Marks: 60] [50 classes]

- Concept of neighbourhood of a point in \mathbb{R}^n ($n > 1$), interior point, limit point, open sets and closed sets in \mathbb{R}^n ($n > 1$).
- Functions from \mathbb{R}^n ($n > 1$) to \mathbb{R} , limit and continuity of functions of two or more variables. Partial derivatives, related mean value theorem, sufficient condition for continuity. Differentiability, sufficient condition for differentiability.
- Directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes.
- Partial derivatives of higher order, sufficient condition for equality of mixed order partial derivatives (Schwarz's and Young's theorems), differentials of higher orders, total differential for function of functions, Chain rule for one and two independent parameters.
 - Euler's theorem on homogeneous functions of two and three variables, change of variables – simple problems. Taylor's theorem of two variables.
- Implicit functions, statement of the existence theorem, derivative of implicit functions – simple problems. Jacobians – elementary properties (statements only) and simple problems.
- Extrema of functions of two variables, constrained optimization problems, method of Lagrangian multipliers for two variables.
- Multiple integral: Concept of upper sum, lower sum, upper integral, lower integral and double integral (no rigorous treatment is needed). Statement of existence theorem for continuous functions.
- Iterated or repeated integral, Statement of Fubini's theorem. Change of order of integration. Areas of plane regions.
- Triple integral. Cylindrical and spherical coordinates.

- Change of variables in double integrals and triple integrals. Transformation of double and triple integrals (problemsonly).
- Determination of volume and surface area by multiple integrals (problems only).
- Differentiation under the integral sign, Leibniz's rule (problemsonly).

Group B: Partial Differential Equations - I

[Marks: 15][10 classes]

- Definition, order and degree of PDE, classification of PDE (linear, quasilinear, semilinear and nonlinear), derivation of partial differential equations (by elimination of arbitrary constants/functions). Examples of PDEs that are central to the study of different problems in science and technology (e.g. Heat equation, Wave equation, Laplace equation, KDV equation).
- First order equations: Solution of quasilinear equations, Lagrange's method of solution. Cauchy problem for quasilinear PDE, The method of characteristics, method of characteristics for linear, semilinear equations; Solution via method of characteristics; Local existence and uniqueness theorem (statement and examples).
- Nonlinear first order partial differential equations, Charpit's general method of solution.

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MATH-H-CC 8-4-TH

Group Theory – II and Ring Theory – I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A : Group Theory- II

[Marks: 40] [32 classes]

- Normal subgroup and its properties. Quotient group. Group homomorphisms, properties of homomorphisms, correspondence theorem and one-one correspondence between the set of all normal subgroups of a group and the set of all congruences on that group, Cayley's theorem, properties of isomorphisms. First, Second and Third isomorphism theorems.
- Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups.
- External direct product and its properties, the group of units modulo n as an external direct product, internal direct product, converse of Lagrange's theorem for finite abelian group, Cauchy's theorem for finite abelian group.

Group B: Ring Theory- I

[Marks:35] [28 classes]

- Definition and examples of rings, properties of rings, subrings, necessary and sufficient condition for a nonempty subset of a ring to be a subring, integral domains and fields, subfield, necessary and sufficient condition for a nonempty subset of a field to be a subfield, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms. First, Second and Third isomorphism theorems, Correspondence theorem, congruence on rings, one-one correspondence between the set of ideals and the set of all congruences on a ring.

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MATH-H-CC 9-5-TH

Probability and Statistics

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group – A: Probability

[Marks: 45][35 classes]

- Random experiment, equally likely outcomes, Sample space, Events, σ -field, Probability as a set function, Probability axioms, Probability space; Conditional probability, The multiplication rule, The law of total probability and Bayes' theorem; Independence of events and trials; Joint probability, Bernoulli trial and binomial law, Poisson approximation of binomial law;
- Real random variables (discrete and continuous), distribution function of a random variable, Properties of distribution function, Probability mass / density functions and properties;
Discrete distributions: Binomial, Poisson;
Continuous distributions: Uniform, Normal, Exponential;
Transformation of a random variable;
Mathematical expectation, Mean, Variance, Moments, Quantiles, Skewness, Kurtosis, Median, Mode;
Moment generating function, Characteristic function;
- Multivariate random variables, Joint distribution of discrete and continuous random variables and their properties, Joint probability mass / density functions, Marginal and Conditional distributions, Independent random variables; Conditional expectations, Expectation of function of two random variables, Moments, Covariance, Correlation coefficient, linear regression for two variables, regression curves;
Bivariate normal distribution;
Distribution of the sums of independent discrete / continuous random variables, Product of two random variables;
Chi-square, t and F-distributions;
- Chebyshev's inequality, Convergence in Probability, Statement of weak law of large numbers and strong law of large numbers; Statement of Central limit theorem;
Statement of De Moivre Laplace limit theorem, Normal approximation of the binomial distribution;
Statement of Uniqueness theorem of Characteristic functions.

Group – B: Statistics

[Marks: 30][25 classes]

- Populations and Samples, Random Sample Sampling and Sampling Distributions, Distribution of the sample, Simple random sampling with and without replacement, Sample Statistic, Sample characteristics - Sample moments, Sample variance, Sampling from the normal distributions;
- Estimation of parameters: Point estimation, Interval Estimation, Mean-squared error, Properties of good estimators - unbiasedness, consistency, sufficiency, Minimum-Variance Unbiased Estimator (MVUE), Unbiased estimators for expectation and variance;
- Method of Maximum likelihood: The maximum likelihood principle, Likelihood function and Loglikelihood function, Maximum likelihood estimators for discrete and continuous models, Properties of maximum likelihood estimators;
- Bivariate frequency Distribution: Bivariate data, Correlation and covariance, Linear Regression, principle of least squares and fitting of polynomials and exponential curves.
- Confidence intervals: General principle; Confidence intervals for the mean of Normal population-for known variance and unknown variance; Confidence interval for variance of Normal population;
- Statistical hypothesis: Simple and composite hypotheses, null hypotheses, alternative hypotheses, one sided and two-sided hypotheses, The critical region and test statistic, type I error and type II error, level of significance, Power function of a test, most powerful test, Neyman-Pearson lemma (Statement only), Likelihood-ratio tests; Tests on the Mean of a Normal Distribution, Variance Known; Tests on the Mean of a Normal Distribution, Variance unknown; Tests on a Population Proportion, Chi-square test for goodness of fit.

References

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MATH-H-CC 10-5-TH

Ring Theory - II and Linear Algebra - I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group – A: Ring Theory - II

[Marks: 40][32 classes]

- Principal ideal domain, principal ideal ring, prime element, irreducible element, greatest common divisor (gcd), least common multiple (lcm), expression of gcd, examples of a ring R and a pair of elements $a, b \in R$ such that $\gcd(a, b)$ does not exist, Euclidean domain, relation between Euclidean domain and principal ideal domain.
- Polynomial rings, division algorithm and consequences, factorization domain, unique factorization domain, irreducible and prime elements in a unique factorization domain, relation between principal ideal domain, unique factorization domain, factorization domain and integral domain, polynomial ring over unique factorization domain, Eisenstein criterion and unique factorization in $\mathbb{Z}[x]$.

- Ring embedding and quotient field, regular rings and their examples, properties of regular ring, ideals in regular rings.

Group –B: Linear Algebra - I

[Marks: 35][28 classes]

- Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Subspaces of \mathbb{R}^n . Dimension of subspaces of \mathbb{R}^n . Geometric significance of subspace up to \mathbb{R}^3 . Four fundamental subspaces associated with a matrix. The dimension of the solution space of $Ax = 0$ and the rank of A. Full rank factorization, rank inequalities, Sylvester's inequality.
- Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, change of coordinate matrix. Algebra of linear transformations. Isomorphisms. Isomorphism theorems, invertibility and isomorphisms. Eigen values, eigen vectors and characteristic equation of a matrix (over C). Cayley-Hamilton theorem and its use in finding the inverse of a matrix.

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MATH-H-CC 11-5-TH

Riemann Integration and Series of Functions

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group – A: Riemann Integration

[Marks: 50][40 classes]

Riemann integration[32 classes]

- Partition of a closed and bounded interval and refinement of a partition. Upper Darboux sum $U(P, f)$ and lower Darboux sum $L(P, f)$ and associated results. Upper integral and lower integral. Darboux's theorem. Darboux's definition of integration over a closed and bounded interval. Riemann's definition of integrability. Equivalence with Darboux's definition of integrability (statement only). Necessary and sufficient condition for Riemann integrability.

- Concept of negligible set (or zero set) defined as a set covered by countable number of open intervals sum of whose lengths is arbitrary small. Examples of negligible sets: any subset of a negligible set, finite set, countable union of negligible sets. A bounded function on closed and bounded interval is Riemann integrable if and only if the set of points of discontinuity is negligible (Statement only). Example of Riemann integrable functions.

- Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions. Properties of Riemann integrable functions arising from the above results.

- Function defined by definite integral $\int_a^x f(t)dt$ and its properties. Antiderivative (primitive or indefinite integral).

- Fundamental theorem of Integral Calculus. First Mean Value theorem of integral calculus. Weierstrass's & Bonnet's form of second mean value theorems (statement only).

Improper integral [8 classes]

- Range of integration, finite or infinite. Necessary and sufficient condition for convergence of improper integral in both cases. Cauchy's principal value of improper integral.

- Tests of convergence: Comparison and μ -test. Absolute and non-absolute convergence and inter-relations. Statement of Abel's and Dirichlet's test for convergence of the integral of product of two functions.

- Convergence and working knowledge of Beta and Gamma function and their interrelation (statement only) $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$, $0 < n < 1$, to be assumed in computation of the integrals $\int_0^{\pi/2} \sin^n x dx$, $\int_0^{\pi/2} \cos nx dx$, $\int_0^{\pi/2} \tan^n x dx$, when they exist (using Beta and Gamma function).

Group B: Series of Functions

[Marks: 25][20 classes]

- Sequence of functions defined on a set, Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Weierstrass's M-test. Boundedness, continuity, integrability and differentiability of the limit function of a sequence of functions in case of uniform convergence.
- Series of functions defined on a set, Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Weierstrass's M-test. Passage to the limit term by term. Boundedness, continuity, integrability, differentiability of a series of functions in case of uniform convergence. Dini's theorem.
- Power series: Fundamental theorem of power series. Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of power series. Properties of sum function. Differentiation and integration of power series. Abel's limit theorems. Uniqueness of power series having sum function.
- Fourier series: Trigonometric series. Statement of sufficient condition for a trigonometric series to be a Fourier series. Fourier coefficients for periodic functions defined on $[-\pi, \pi]$. Statement of Dirichlet's condition of convergence. Statement of Fourier's theorem on sum of Fourier series.

References

- [1] R.G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
- [2] G. G. Bilodeau, P. R. Thie, G.E. Keough, An Introduction to Analysis, 2nd Ed., Jones & Bartlett, 2010.
- [3] B. S. Thomson, A. M. Bruckner and J. B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.
- [4] S.K. Berberian, A First Course in Real Analysis, Springer Verlag, New York, 1994.

- [5] T. Apostol, Mathematical Analysis, Narosa Publishing House, 1969.
 [6] R. Courant and F. John, Introduction to Calculus and Analysis, (Volumes I & II), Springer-Verlag, New York, Inc., 1998.
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 [9] T. Tao, Analysis I, Hindustan Book Agency, 2006.
 [10] R. R. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing, 2020.
 [11] H. R. Beyer, Calculus and Analysis, Wiley, 2010

MATH-H-CC 12-5-TH Mechanics-II

Full Marks: 100 (Theory: 75 and Tutorial: 25)

(60classes)

Statics-II:

- **Friction:** Laws of static friction, Limiting friction, Angle of friction and Cone of friction. Positions of equilibrium of a particle constrained to rest on a (i) rough plane curve and (ii) rough surface. [4 classes]
- **Virtual work:** Degrees of Freedom, Constraints, Virtual Displacement, Virtual Work, Workless Constraints, Forces which do not appear in the equation of virtual work, Forces which appear in the equation of virtual work, Principle of virtual work for any system of coplanar forces acting on a rigid body and deduction of conditions of equilibrium from the Principle of virtual work. [6 classes]
- **Stable and unstable equilibrium:** Field of forces, Conservative field, Potential energy of a system, Concepts of Stable, Unstable and Neutral equilibrium, Energy test of stability for a system having one degree of freedom, Stability when gravity is the only external force, Condition of stability of equilibrium of two heavy bodies resting one upon another, the bodies being rough enough to prevent sliding, [6 classes]
- **Arbitrary force system in three dimensions:** Axis of a couple, Resultant of any number of couples acting on a rigid body, Reduction of a system of forces acting on a rigid body, Equilibrium equations, Reduction to wrench intensity and pitch of a wrench, Poinso't's central axis, Equation of the central axis of a given system of forces, Invariants of a given system of forces. [6 classes]

Dynamics of a Particle-II:

- Stability of nearly circular orbits, Disturbed orbits, Motion of a particle on rough curve, Expressions for components of velocity and acceleration referred to a set of rotating axes, Motion of a particle of varying mass including problems of mass addition (Rain-drop Problem) and mass reduction (Rocket Problem). [8 classes]

- **Dynamics of a system of particles:** General theorems (Emphasis should be given on theoretical discussion only in this part): Configuration of a mechanical system and its degrees of freedom, External forces, Internal forces and two assumptions connected with these forces, Mass of a system, Centre of mass of a system and its motion, Linear momentum of a system and principle of conservation of linear momentum, Angular momentum of a system about a point and an axis, Angular momentum principle about the centre of mass, Conservation of angular momentum about a point and an axis, Kinetic energy(K.E.) of a system, The energy principle and Conservation of energy. [4 classes]

Dynamics of rigid body:

- Vector angular velocity and its existence, particle velocities in a rigid body. [2 classes]

- Moments and Products of Inertia, Moment of inertia of a body about any line through the origin of a coordinate frame, Radius of gyration, Equipomental systems, Principal axis and Momental ellipsoid, theorems of parallel and perpendicular axes(statements only). [4 classes]

- **General motion:** Deduction of the equations: $M \frac{d\vec{V}}{dt} = \vec{F}$, $\frac{d\vec{L}_G}{dt} = \vec{K}_G$ from linear and angular momentum principle, Deductions of equations of motions from D'Alembert's Principle, Independence of the motion of centre of inertia and the motion relative to the centre of inertia, Angular momentum of a rigid body and the kinetic energy of a rigid body rotating about a fixed axis, Motion of a rigid body about a fixed axis, Compound pendulum, Interchangeability of the point of suspension and centre of oscillation. [10 classes]

- **Motion of a rigid body in two dimensions:** Equations of motion of a rigid body in two dimensions in the form $M \frac{dV_x}{dt} = F_x$, $M \frac{dV_y}{dt} = F_y$, $I \frac{d\omega}{dt} = K_G$. Expressions for K.E. and angular momentum about the origin, Condition of pure rolling and sliding. [6 classes]

- **Motion under impulsive forces:** Equation of motion for impulsive forces for two dimensions, Statements of the conservation of linear and angular momentum. Problems of impulse applied to a free rod and a rod constrained to rotate about a fixed axis. [4 classes]

References

[1] D. Chernilevski, E. Lavrova and V. Romanov, Mechanics for Engineers, MIR Publishers, 1984.
 [2] F. Chorlton, Textbook of Dynamics, CBS Publishers, 2002 (2nd Edition).
 [3] R. D. Gregory, Classical mechanics, Cambridge University Press, 2006.
 [4] D.T. Greenwood, Principle of Dynamics, Prentice-Hall, 1988 (2nd Edition).
 [5] D. Kleppner and R. Kolenkow, An Introduction to Mechanics, Cambridge

University Press, 2010.

[6] S.L. Loney, An Elementary Treatise on the Dynamics of Particle and of Rigid

Bodies, Cambridge University Press, 1913.

[7] S.L. Loney, An Elementary Treatise on Statics, Cambridge University Press, 1917 (2nd Edition).

[8] A.S. Ramsey, Dynamics (Part I & II), CBS Publishers, 2002 (2nd Edition).

[9] J.L. Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, 1959 (3rd Edition).

[10] S. Timoshenko and D. H. Young, Engineering Mechanics, McGraw-Hill, 2017 (5th Edition).

[11] S. T. Thornton and J. B. Marion, Classical Dynamics of Particles and Systems, Cengage Learning, 2017 (5th Edition).

MATH-H-CC 13-6-TH

Metric Space and Complex Analysis-I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group-A: Metric Space

- Definition and examples of metric spaces. Open ball. Open set. Closed set defined as complement of open set. Interior point and interior of a set. Limit point and closure of a set. Boundary point and boundary of a set. Properties of interior, closure and boundary. Bounded set and diameter of a set. Distance between two sets. Subspace of a metric space.
- Continuous mappings. Uniform continuity. Isometry.
- Convergent sequence. Sequential criterion of continuity. Cauchy sequence. Every convergent sequence is Cauchy and bounded, but the converse is not true. Completeness. Cantor's intersection theorem. \mathbf{R} is a complete metric space. \mathbf{Q} is not complete. Completion of metric spaces. Nowhere dense set, Baire Category theorem.
- Compactness, Sequential compactness. Concept of Lebesgue number. Lebesgue covering lemma. Equivalence of compactness and sequential compactness of a metric space. Heine-Borel theorem in \mathbf{R} . Finite intersection property, continuous functions on compact sets. Total boundedness and its properties. A metric space is compact if and only if the space is complete and totally bounded (statement only).

- Concept of connectedness and some examples of connected metric spaces, connected subset of \mathbf{R}, \mathbf{C} .
- Contraction mappings, Banach Fixed point theorem and its applications.

Group - B : Complex Analysis - I

- Stereographic projection. Regions in the complex plane. Limits, limits involving the point at infinity. Continuity of functions of a complex variable.
- Derivatives, differentiation formulae, Cauchy-Riemann equations, sufficient conditions for differentiability. Analytic function, entire function, exponential function, logarithmic function, trigonometric functions, hyperbolic functions.
- Möbius transformation and its properties.
- Power series: Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of power series. Analytic functions represented by power series. Uniqueness of power series.
- Harmonic functions and its properties, Milne Thompson method.

References

- [1] S. Kumaresan, Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.
- [2] P. K. Jain and K. Ahmad: Metric Spaces, Narosa Publishing House.
- [3] M. N. Mukherjee, Elements of Metric Space, Academic Publisher.
- [4] Satish Shirali & Harkrishan L. Vasudeva, Metric Spaces, Springer-Verlag, London, 2006.
- [5] H. A. Priestley, Introduction to Complex Analysis, Second Edition, Oxford University Press, 2003.
- [6] Robert E. Greene & Steven G. Krantz, Function Theory of One Complex Variable, Third Edition, Graduate Studies in Mathematics, Volume 40, AMS, 2006.
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- [8] R.P. Agarwal, K. Perera and S. Pinelas; An Introduction to Complex Analysis; Springer-Verlag, 2011.
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- [10] J. B. Conway; Functions of One Complex Variable; Narosa Publishing, New Delhi, 1973.
- [11] T. W. Gamelin; Complex Analysis; Springer International Edition, 2001.
- [12] S. Lang; Complex Analysis (Fourth edition); Springer-Verlag, 1999.
- [13] R. Narasimhan; Complex Analysis in one variable; Birkhauser, Boston, 1984.
- [14] S. Ponnusamy; Foundations of Complex Analysis, second edition; Narosa Publishing, New Delhi, 2005.
- [15] H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.
- [16] E. M. Stein and R. Shakarchi; Complex Analysis; Princeton University Press, Princeton, New Jersey, 2003.
- [17] James Ward Brown and Ruel V. Churchill, Complex Variables and Applications, 8th Ed., McGraw – Hill International Edition, 2009.
- [18] Joseph Bak and Donald J. Newman, Complex Analysis, 2nd Ed., Undergraduate Texts in Mathematics, Springer-Verlag Inc., New York, 1997.

MATH-H-CC 14-6-TH

Multivariate Calculus-II & Application of Calculus

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group - A: Multivariate Calculus

[Marks: 50][40 classes]

- Functions from R^n ($n > 1$) to R^m ($m \geq 1$). Continuity and differentiability of such functions. Jacobian matrix. A function $f: R^n \rightarrow R^m$ is differentiable if and only if it is every comp

one is differentiable,

- Inverse function theorem (statement only), implicit function theorem (statement only).
- Level sets, tangent spaces.
- Double integration over rectangular region, double integration over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals. Differentiation under the integral sign, Leibniz's rule.
- Definition of vector field, divergence and curl. Line integrals, applications of line integrals: mass and work. Fundamental theorem for line integrals, conservative vector fields, independence of path.
- Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stoke's theorem, The Divergence theorem.

Group-B: Applications of Calculus

[Marks: 25][20 classes]

- Curvature, concavity and points of inflection, envelopes (Cartesian coordinates only), rectilinear asymptotes (Cartesian & parametric form only), curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves.

References

- [1] Michael Spivak; Calculus on Manifolds; Westview Press; 1998.
- [2] G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson Education, Delhi, 2005.
- [3] M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
- [4] E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer (SIE), 2005.
- [5] James Stewart, Multivariable Calculus, Concepts and Contexts, 2nd Ed., Brooks/Cole, Thomson Learning, USA, 2001
- [6] T. Apostol, Mathematical Analysis, Narosa Publishing House.
- [7] Courant and John, Introduction to Calculus and Analysis, Vol II, Springer

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[9] Horst R.Beyer,CalculusandAnalysis,Wiley,2010.

MATH-H-CC 15-6-TH

Numerical Analysis

Full Marks: 100 (Theory: 75 and Practical: 25)
(Theory: 45 classes + Practical: 15 classes)

- **Representation of numbers and errors in Numerical Analysis:** Machine Numbers - floating point and fixed point, Sources of Errors, Rounding of numbers, Absolute and Relative Error, Significant digits and Error Propagation in machine arithmetic operations. Numerical Algorithms - stability and convergence.

- **Approximation:** Classes of approximating functions, polynomial approximation, The Weierstrass' polynomial approximation theorem (statement only).

- **Interpolation:** *Prerequisite* -Divided difference:Definition of n -th divided difference $f[x_0, x_1, \dots, x_n]$ and simple properties. Finite difference operators $\Delta, \nabla, E, \mu, \delta$ -definitions and simple relations among them.

Principle of interpolation, polynomial interpolation, existence and uniqueness of interpolating polynomial. Lagrange's interpolation polynomial, Newton's divided difference interpolation polynomial, Error in polynomial interpolation. Interpolation based on evenly spaced points: Difference table, Newton's forward and backward interpolation formulae, error terms. Central Interpolation: Statements of Stirling's and Bessel's formulae. Different interpolation zones, Error estimation. Statement of Hermite's interpolation formula, its uniqueness and error (only statement).

- **Numerical differentiation:** Basic principle, Methods based on Lagrange's, Newton's forward and backward interpolation formulae.

• **Numerical Integration:** Newton-Cotes formula (deduction of closed type), Trapezoidal rule and Simpson's $1/3^{\text{rd}}$ rule - composite form. Statement of Weddle's rule and composite Weddle's rule. Error formulae (only statements), Degree of precision.

• **Numerical solution of non-linear equations:** Bisection method, Secant method, Regula-falsi method, fixed point iteration, Newton-Raphson method. Condition of convergence (if any), Order of convergence, Rate of convergence of these methods. Advantages and disadvantages of the methods.

• **Numerical solution of system of linear equations:** Direct methods-Gaussian elimination and Gauss-Jordan methods, partial pivoting strategies, operational count. Iterative methods- Gauss-Jacobi method, Gauss-Seidel method and their convergence analysis, Advantages and disadvantages of the methods.

• **Matrix inversion** by Gaussian elimination.

• **Algebraic eigenvalue problem:** The Power method.

• **Numerical solution of initial value problems for ordinary differential equations:** Single-step difference equation methods—idea of local truncation error, consistency and convergence. Euler's method, Modified Euler method, Higher order Taylor methods, Runge-Kutta method of order four, Picard's method of successive approximation.

Problems for Practical (using C programming language)

1. Interpolation by Lagrange's formula
2. Interpolation by Newton's forward formula
3. Interpolation by Newton's backward formula
4. Integration by composite Trapezoidal rule
5. Integration by composite Simpson's $1/3^{\text{rd}}$ rule
6. Integration by composite Weddle's rule
7. Solution of equation by Bisection method
8. Solution of equation by Newton-Raphson method
9. Solution of equation by Regula-falsi method
10. Solution of a system of linear equations by Gaussian elimination method
11. Solution of a system of linear equations by Gauss-Seidel method

12. Matrix inversion by Gaussian elimination method
13. Finding largest eigen-pair of a matrix Power method
14. Solution of ordinary differential equation by modified Euler method
15. Solution of ordinary differential equation by fourth order Runge-Kutta method

References

- [1] Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
- [2] M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, 6th Ed., New age International Publisher, India, 2007.
- [3] C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Pearson Education, India, 2008.
- [4] Uri M. Ascher and Chen Greif, A First Course in Numerical Methods, 7th Ed., PHI Learning Private Limited, 2013.
- [5] John H. Mathews and Kurtis D. Fink, Numerical Methods using Matlab, 4th Ed., PHI Learning Private Limited, 2012.
- [6] James B. Scarborough, Numerical Mathematical Analysis, John Hopkins University Press, 1966
- [7] K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons, 1978.
- [8] A.Gupta and S. Bose: An Introduction to Numerical Analysis, Academic Publishers, 2009.
- [9] R. L. Burden and J. D. Faires, Numerical Analysis, 9th Edition, Thomson.

MATH-MD-CC 5-4-TH Advanced Calculus

**Full Marks: 100 (Theory: 75 and Tutorial: 25)
(60classes)**

Group A

[Marks: 20] [16 classes]

Sequence of real numbers: Definition of bounds of a sequence and monotone sequence. Limit and convergence of a sequence. Statements of limit theorems.

Convergence and divergence of monotone sequences and applications. Convergence of r^n , $r^{\frac{1}{n}}$ and $n^{\frac{1}{n}}$ in particular. Statement of Cauchy's general principle of convergence and its applications.

Infinite series of constant terms: Convergence and Divergence (definitions and examples). Cauchy's principle as applied to infinite series (application only). Series of positive terms: Statements of Comparison test, Limit form of Comparison Test, D. Alembert's Ratio test. Cauchy's root test and Rabbe's test: Applications, Alternating series: Statement of Leibnitz's test and its applications.

Group B

[Marks: 25] [20 classes]

Real-valued functions defined on an interval: Limits of functions ($\varepsilon - \delta$ approach), Cauchy's criterion of existence of limit (statement only). Algebra of limits (statements only), Continuity of a function at a point and on an interval, Properties of continuous functions on closed and bounded intervals without proof (Boundedness and Intermediate value property) Statement of existence of inverse function of a strictly monotone function and its continuity.

Derivative: Statement of Rolle's theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy as a consequence of Rolle's theorem. Geometrical interpretation of Lagrange's Mean value Theorem, Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders, Taylor's and Maclaurin's Infinite series, Expansion of the functions e^x , $\sin x$, $\cos x$, $(1+x)^n$ and $\log(1+x)$ [with restrictions wherever necessary and assuming $R_n \rightarrow 0$]

Group C

[Marks: 30] [24 classes]

Multivariate Calculus: Real-valued Functions of two and three real variables: Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables, Partial derivatives, Differentiability, sufficient condition for differentiability (statement only), Chain rule for two independent variables, Higher order partial derivatives up to second order, Statement of Schwarz's Theorem for equality of mixed partial derivatives, Euler's theorem on homogeneous functions of two variables. Derivative of Implicit functions of two variables (existence assumed).

References:

- [1] G.B. Thomas and R.L. Finney, Calculus, 14th Ed., Pearson Education, Delhi, 2018.
- [2] R. Courant and F. John, Introduction to Calculus and Analysis (Vol. I & II), Springer- Verlag, New York, Inc., 1998.
- [3] T. Apostol, Calculus, Volumes I and II, John Wileyand Sons, 1969.
- [4] S. Lang, A First course in Calculus, Springer, 1986.
- [5] S. C. Malik and S. Arora, Mathematical Analysis, New Age International Publishers, 4e, 2014.

MATH-MD-CC 6-4-TH

Statistics and Numerical Analysis

Full Marks: 100 (Theory: 75 and Tutorial: 25)
(60classes)

Group A: Statistics

[50 Marks] [40 classes]

Probability Theory: Theorems on Total Probability, Conditional probability and Multiplication theorem, Bayes' Theorem (Application only). Independence of events, Related Problems.

Compound experiment, Independent trials, Bernoulli's trials, Binomial law.

Probability Distribution: Random Variables, Probability Distribution function, Properties of probability distribution function, Discrete and continuous distribution, Probability mass and probability density function, Some important probability distributions and their properties – Binomial, Poisson, Uniform and Normal, Related problems.

Two-dimensional random variables and bivariate distribution (discrete and continuous), Marginal distribution, Bivariate Uniform and Normal distributions, Related problems.

Mathematical Expectation: Definition of mathematical expectation, Mean, Variance, Standard Deviation, Moments, Theorems on mathematical

expectation(statementonly), Standardized random variate, Mean, Variance and standard deviation of Binomial, Poisson and Normal distributions.

Mathematical Expectation in bivariate distribution, Moments, Covariance, Correlation coefficient, $E(X + Y) = E(X) + E(Y)$, $E(XY) = E(X).E(Y)$ for independent variates.

Elements of Statistical Methods: Measure of Central tendency: Arithmetic Mean, Geometric Mean, Harmonic Mean, Median and Mode (their advantages and disadvantages), Relation between Mean, Median and Mode

Measures of Dispersion: Range, Quartile Deviation, Mean Deviation, Variance / Standard Deviation

Moments, Raw moments and Central moments and relation between them, Effect of change of origin and change of scale on moments.

Correlation and Regression: Bivariate data, Scatter Diagram, Correlation coefficient-its determination and properties. Regression lines of y on x and x on y – their deductions and properties.

Sampling Theory: Meaning and objects of sampling, Parameter and Statistic, Sampling distribution of a statistic, Methods of drawing random sample, Mean and Variance of sample mean, Basic idea of some distributions used in Sampling Theory - (i) Standard Normal distribution, (ii) Chi-square distribution (iii) Student's t-distribution (iv) Snedecor's F-distribution, Related problems.

Statistical Inference: Estimation of Parameters, Unbiased estimator, Consistent estimator, Sample mean is an unbiased estimate of population mean, Sample variance is a biased estimator of population variance, Point estimation, Interval estimation, Method for finding Confidence Intervals, Confidence intervals for m of Normal (m, σ) population when σ is known and when σ is unknown, Statistical Hypothesis – Null Hypothesis and Alternative Hypothesis, Critical Region, Type I and II error, Level of significance, Related simple problems.

(Note: Emphasis should be given on definitions and problem solving)

Group B: Numerical Analysis

[25 Marks] [20 classes]

Approximate numbers: Significant figures, Rounding off of numbers. Errors – Absolute, Relative and Percentage.

Operators - Δ , ∇ and E (Definitions and some relations among them).

Interpolation: Problem of interpolation, Equi-spaced arguments, Difference Table, Deduction of Newton's Forward Interpolation Formula, remainder term (expression only). Newton's Backward interpolation Formula (Statement only) with remainder term. Unequally-spaced arguments, Lagrange's Interpolation Formula (Statement only), Related problems.

Numerical Integration: Trapezoidal rule and Simpson's 1/3-rd rule with geometrical interpretation, Related problems.

Numerical solution of nonlinear equations: To find a real root of an algebraic or transcendental equation. Location of root (tabular method), Bisection method, Newton-Raphson method with geometrical interpretation, Related problems.

Numerical solution of system of linear equations: Gauss elimination method using partial pivoting for solution of system of three linear equations in three unknowns.

(Note : Emphasis should be given on problems)

References

- [1] W. Feller, An introduction to Probability Theory and its Applications, Volume 1, 3e, Wiley, 1968.
- [2] R. V. Hogg, J. W. McKean and A. T. Craig, Introduction to Mathematical Statistics, Pearson Education, Asia, 2007.
- [3] S. Ross, Introduction to Probability Models, 9th Ed., Academic Press, Indian Reprint, 2007.

- [4] A. M. Goon, M. K. Gupta and B. Dasgupta, Fundamental of Statistics, World Press, Vol 1, 2013 & Vol 2, 2016.
- [5] A. Gupta, Groundwork of Mathematical Probability and Statistics, Academic Publishers, 1983.
- [6] B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
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MATH-MD-CC 7-5-TH

Mathematical Methods

Full Marks: 100 (Theory: 75 and Tutorial: 25)
(60classes)

Group A

[Marks: 25] [20 classes]

Sequence and Series of functions: Concept of Point-wise and Uniform convergence of sequence of functions and series of functions defined on intervals with special reference of Power Series, Statement of Weierstrass' M-Test for Uniform convergence of sequence of functions and of series of functions, Simple applications. Statements of important properties like boundedness, continuity, differentiability of the limit function of uniformly convergent sequence of functions and of the sum function of uniformly convergent series of functions, Convergence of Power Series, Radius of convergence, Statement of Abel's Theorem on Power Series and region of convergence of Power Series, Simple problems.

Group B

[Marks: 35] [28 classes]

Application of differential calculus: Tangents and Normals, Pedal equations, Curvature, radius of curvature of plane curves, Rectilinear asymptotes (Cartesian only), Envelopes (Cartesian only), Concavity, Convexity, Point of inflection (Cartesian only), Definition and examples of singular points (viz. Node, Cusp, Isolated point) (Cartesian only).

Maxima and minima of functions: Maxima and Minima of functions involving one and two variables, Lagrange's Method of undetermined multipliers of functions involving two variables (Problems only).

Group C

[Marks: 15] [12 classes]

Fourier series on $[-\pi, \pi]$: Periodic function, Determination of Fourier coefficients, Statement of Dirichlet's conditions (improperness excluded) for convergence and statement of the theorem on convergence and sum of Fourier series, Sine and Cosine series.

Laplace Transform and its application to ordinary differential equation: Laplace Transform and Inverse Laplace Transform, Statement of Existence theorem, Statements of elementary properties of Laplace Transform and its Inverse, Application to the solution of ordinary differential equation of second order with constant coefficients.

References:

- [1] D. Widder; Advanced Calculus; Prentice Hall, 1947.
- [2] A. E. Taylor and W. Robert Mann; Advanced Calculus (3rd Edition); John Wiley & Sons, Inc, 1983.
- [3] R. C. Wrede and M. Spiegel; Advanced Calculus, (Schaum's outline series); McGraw – Hill, 2020.
- [4] G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson Education, Delhi, 2005.
- [5] M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) P. Ltd. (Pearson Education), Delhi, 2007.
- [6] H. Anton, I. Bivens and S. Davis, Calculus, 7th Ed., John Wiley and Sons (Asia) P. Ltd., Singapore, 2002.
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- [9] J. Edwards, An Elementary treatise on Differential Calculus, Radha Publishers, 1992.
- [10] J. M. H. Olmstead, Advanced Calculus, Prentice Hall, 1961.

MATH-MD-CC 8-6-TH

Discrete Mathematics

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A

[Marks: 30] [24 classes]

Graph Theory:

Definition and examples of graphs, Walks, Trails, Paths, Circuits and cycles, Eulerian circuits and paths. Eulerian graphs, example of Eulerian graphs. Hamiltonian cycles and Hamiltonian graphs.

Definition of Trees and their elementary properties. Definition of Planar graphs, Kuratowski's graphs.

Group B

[Marks: 45] [36 classes]

Application of Congruences: Congruence classes, addition and multiplication of congruence classes, Fermat's little theorem, Euler's Theorem, Wilson's theorem (statements only) and some applications. Divisibility tests by 2, 3, 4, 5, 7, 9, 11, 13 (Statements of relevant results, thrust will be on working out problems), Check Digits in Ten Digit International Standard Book Number (ISBN), Universal Product Code (UPC), VISA and MASTER card (Statements of relevant results and Problems only), Formation of Round Robin Tournament Table using congruence of integers (Technique and Problems only).

Combinatorics, Recurrence Relations and Generating functions: Statement of Fundamental counting principle, Recurrence Relations, Solution of recurrence relations by the method of Iteration (Consideration of problems of Rabbit on an island, The tower of Hanoi), Linear difference equations up to second order with constant coefficients.

Boolean Algebra: Boolean Algebra (Definition, examples and basic properties), Boolean functions, Boolean polynomials, minimal forms of Boolean polynomials - CNF & DNF, Switching circuits and minimization of switching circuits using Boolean algebra, Logic Gates (Definition and examples).

References:

- [1] R. J. Wilson: Introduction to Graph Theory; 4th edition, Pearson, 2007.
- [2] E. G. Goodaire and M. M. Parmenter: Discrete Mathematics with Graph

- Theory; 3rd Edition, Pearson India Education Services Pvt. Ltd., 2015.
- [3] J. Clark and D. Holton: A first look at Graph Theory, World Scientific Publishing Co. Pvt. Ltd.(Allied Publishers Ltd.) 1991.
- [4] N. Deo: Graph Theory with applications to Engineering and Computer Science, Prentice-Hall of India, 2000.
- [5] K. H. Rosen: Discrete Mathematics and its applications with combinations and Graph Theory (7th Edition), Mc-Graw Hill Education (India) Pvt. Ltd., 2011.
- [6] F. Harary; Graph Theory; Narosa Publishing House, New Delhi, 2001.
- [7] M. K. Sen and B.C. Chakraborty, Introduction to Discrete Mathematics, 4th Edition, Books & Allied Pvt. Ltd., 2019.